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EMPIRICAL BAYES ESTIMATION OF PROPORTIONS IN SEVERAL GROUPS

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A straightforward empirical Bayes approach to the problem of estimating binomial proportions in several similar but not necessarily identical groups is proposed. The approach compares favorably with simlar methods proposed by Jackson (1972) and Novich, Lewis, and Jackson (1973).

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EMPIRICAL BAYES ESTIMATION OF PROPORTIONS IN SEVERAL GROUPS

The problem of estimating binomial proportions in several similar but not necessarily identical groups occurs frequently in psychological and educational settings. A straightforward empirical Bayes approach to this problem using a slight extension of the standard Bayesian method for estimating a single proportion is proposed. Novick, Lewis, and Jackson (1973) suggest a purely Bayesian solution to the problem which uses a root arcsine transformation of the proportions. They contrast their method with a similar approach using the same transformation, due to Jackson (1972) using examples to show that the Bayesian approach is better. This note shows that an improved version of Jackson's approach yields much more satisfactory results than the original, so the advantage of the purely Bayesian approach is questionable. But the revised version of Jackson's approach and the new beta-binomial approach yield practically identical results, so even the need for using the root-arcsine transformation is questionable, except when the proportions are concentrated near zero or one.

EMPIRICAL BAYES ESTIMATION OF PROPORTIONS IN SEVERAL GROUPS

I. Introduction

The problem of estimating binomial proportions in several similar but not necessarily identical groups occurs frequently in psychological and educational settings. Standard practice suggests two conflicting ways to deal with the problem. One might stress the similarities between the groups and use the overall proportion as a common estimate for each group proportion. This option makes sense if the variance between the group proportions is so small that it is plausible that it is just due to the variability of sample proportions drawn from the same population. The second possibility stresses the differences between the groups. It assumes that each group is a sample from a population distinct from the other groups and hence uses the sample proportion for each group to estimate its distinct population proportion. The greater the variance between the proportions in the separate groups, the more justified the second option becomes. The typical situation falls in a awkward middle ground where neither way suggested by standard practice seems completely appropriate: it is implausible on the basis of the data that the groups come from identical populations, yet the groups are similar enough that one must view with some skepticism sample proportions which differ markedly from the rest of the groups, particularly when those proportions are based on relatively small samples. This paper concerns efforts to find a middle way between the two approaches just described which can weigh the direct information available in a single sample with the collateral information obtained from distinct but similar samples.

Jackson (1972) proposes a method inspired by the Kelley regression estimate of true score given observed score, from classical test theory. This is a weighted estimator which gives weight $r_{xx'}$ to an individual's observed test

score and weight $1-r_{xx'}$ to the mean score for all individuals, where $r_{xx'}$ is the reliability coefficient. The analogy between the testing situation and the present problem is made by identifying the group proportions with test scores. Actually, the root arcsine transforms of the proportions are used instead because they fit the test-theory assumptions behind the Kelley formula better, being approximately normally distributed with conditional variance, given the true proportion, independent of the value of the true proportion. Observing that the reliability coefficient can be thought of as the ratio of true-score variance to observed score variance reduces the problem of determining how much weight to give to individual scores (or transformed group proportions) to one of estimation of variance components. Jackson suggests simple estimators which lead to easily computed estimates of the proportions.

By specializing results of Lindley and Smith (1972) concerning Bayesian analysis of variance-components in the general linear model, Novick, Lewis, and Jackson (1973) provide a Bayesian rationale for and extension of Jackson's approach. Their extension has both advantages and disadvantages. The main advantage is that the more thorough-going Bayesian approach avoids some nasty problems associated with the estimation of variance components, such as negative estimates of between-groups variance, which can make Jackson's original approach yield unconvincing analyses. The disadvantages are increased computational complexity and some technical problems concerning specification of the prior distribution for transformed proportions.

The problems with the specification of the prior distribution of the transformed proportions derive from the fact that the transformed proportions are bounded between 0 and $\pi/2$. Let γ denote the true transformed proportion for a given group, and let μ_γ and σ_γ^2 denote the mean and variance of the prior distribution of γ . The calculation of posterior distributions is greatly facilitated by assuming that the joint prior distribution of μ_γ and σ_γ^2 has them independently

distributed, with μ_γ uniformly distributed, σ^2_γ distributed as inverse chi-squared, and γ, given μ_γ and σ^2_γ, normally distributed. These assumptions are jointly compatible only if the prior for σ^2_γ is tightly specified to keep the probability of large values of σ^2_γ negligible. This is no problem if a tight specification accurately represents the investigator's prior beliefs. However, Novick, Lewis, and Jackson caution that setting an unrealistically tight prior in order to make the mathematical machinery run can have serious consequences.

If the conditional distribution of γ is to be approximately normal, then there must be room for two standard deviations between μ_γ and the nearest bound. Therefore, the probability of large values of σ^2_γ must be kept small. Of course, the finite range of γ puts a limit on the size of σ^2_γ anyway, but the normality assumption sharpens the bound considerably. Plausible skewed conditional distributions for γ could have values of σ^2_γ much closer to the upper bound. The added constraint that the assumption of normality places on plausible values of σ^2_γ is more stringent for values of μ_γ near 0 or $\pi/2$ than for μ_γ near $\pi/4$, so the assumption that μ_γ and σ^2_γ are independently distributed further exacerbates the problem.

In the Novick, Lewis, and Jackson procedure the estimated proportions for the groups are obtained from the mode of the joint posterior distribution of the γ's. It is necessary to solve a set of simultaneous equations, the Lindley equations, to find this mode. The solution cannot be given in closed form and the process of successive approximation can require many iterations to converge. Sometimes a unique mode does not exist. Novick, Lewis, and Jackson give formulas which can be calculated to check for uniqueness.

It is not clear how critical the problems just discussed are in practice. However, these problems and the increased computational complexity of the procedure do suggest that other ways to modify Jackson's simpler approach which could

avoid its weak points, but retain its computational simplicity, would be welcome. The following sections will show that simply using a better estimator of σ_Y^2 in Jackson's procedure goes far in this direction and that an even simpler approach inspired by the standard Bayesian analysis for a single proportion, which does not require a transformation, is possible. The latter approach is described in the next section. Jackson's approach and the proposed modification of it are discussed in the section after that. Finally, some examples discussed by Novick, Lewis, and Jackson are reexamined in the last section.

2. A Beta-Binomial Solution

The usual Bayesian approach to the estimation of a binomial proportion $\underline{p}$ assumes that the investigator's personal probability distribution for $\underline{p}$ prior to the gathering of data can be represented by a member of the beta family of distributions, the natural conjugate family for the binomial distribution. If we let $\underline{r}'$ and $\underline{n}'$ be the parameters identifying the particular member of this family which represents the investigator's prior belief, then the density of the prior distribution is given by

$$f_\beta(\underline{p}|\underline{r}',\underline{n}') = \frac{1}{B(\underline{r}',\underline{n}'-\underline{r}')}\, \underline{p}^{r'-1}\,(1-\underline{p})^{n'-r'-1} \tag{2.1}$$

where $B(\underline{r}', \underline{n}' - \underline{r}')$ is the complete beta function. The mean of the investigator's prior distribution is $\underline{r}'/\underline{n}'$ and the posterior distribution, given $\underline{r}$ successes in a sample of $\underline{n}$ tries of an experiment carried out to provide further information about $\underline{p}$, is the beta distribution with parameters $\underline{r} + \underline{r}'$ and $\underline{n} + \underline{n}'$. The mean of the posterior distribution, which can be used as a point estimate of $\underline{p}$, is given by

$$\begin{aligned} \hat{\underline{p}} &= \frac{\underline{r}+\underline{r}'}{\underline{n}+\underline{n}'} \\ &= \frac{\underline{n}}{\underline{n}+n'} \cdot \frac{\underline{r}}{\underline{n}} + \frac{\underline{n}'}{\underline{n}+\underline{n}'} \cdot \frac{r'}{\underline{n}'} \ . \end{aligned} \tag{2.2}$$

The second expression for $\hat{p}$ shows that it is a weighted average of the sample proportion, $\underline{r}/\underline{n}$, and the mean of the prior distribution of $\underline{p}$, $\underline{r}'/\underline{n}'$. The sample proportion receives most of the weight if $\underline{n}$ is large relative

to $\underline{n}'$, whereas the overall mean, $\underline{r}'/\underline{n}'$, receives the most weight if $\underline{n}$ is small relative to $\underline{n}'$.

When data from J different groups are available, it is possible for the experimenter to avoid the difficult problem of specifying his or her prior distribution for $\underline{p}$, because the parameters $\underline{r}'$ and $\underline{n}'$ can be estimated from the data (hence the term empirical Bayes). If all J proportions are based on the same number of observations, and the J different $\underline{p}$ parameters are assumed to be a random sample from a population with a beta distribution, then the distribution of the sample proportions is beta-binomial. The parameters of the beta-binomial distribution are the number of observations per group, the number of groups, and the parameters $\underline{r}'$ and $\underline{n}'$ of the underlying beta distribution. By equating the sample mean and variance of the obtained number of successes in each group with theoretical expressions for their expected values in a beta-binomial, one obtains a system of equations which can be solved for $\underline{n}'$ and $\underline{r}'$. The resulting estimators are

$$(2.3) \qquad \hat{\underline{n}}' = \frac{\underline{n}(\underline{n} - 1)\ \underline{p}.(1 - \underline{p}.) - \Delta v_r}{\Delta v_r} ,$$

$$\hat{\underline{r}}' = \underline{p}.\hat{\underline{n}}' ,$$

where $\underline{p}.$ is the mean of the sample proportions and $\Delta v_r = s_r^2 - \underline{n}\ \underline{p}.(1 - \underline{p}.)$ is the difference between the sample variance of the number of successes between groups and the variance one would expect if the average p value had generated the data in all the groups.

Substituting the estimates of $\underline{r}'$ and $\underline{n}'$ given in Equation 2.3 into Equation 2.2 yields a formula for estimating the proportions in each group. These results, which assume that the sample sizes are equal, were given by Maritz (1966). We now propose a generalization which does not require this

assumption. First, let us rewrite the expression for $\hat{\underline{n}}'$ in Equation 2.3 as

$$(2.4) \qquad \hat{\underline{n}}' = \frac{\underline{p}.(1 - \underline{p}.) - \hat{\sigma}_p^2}{\hat{\sigma}_p^2} ,$$

where $\hat{\sigma}_p^2 = \Delta v_r/n\ (n - 1)$ is an unbiased estimate of the variance of the beta distribution generating the $\underline{p}$'s. The idea of estimating $\underline{n}'$ and $\underline{r}'$ by equating the first two sample moments of the distribution of the number of successes with the theoretical values for the beta-binomial breaks down because the sampling distribution is no longer beta-binomial when the $\underline{n}$'s are unequal. Equation 2.4 can be used as the basis for the estimation if we can find a suitable estimator for σ_p^2 . One possible estimator can be obtained from a general theorem concerning the analysis of variance components in an unbalanced one-way random effects model. If we assume the means in such a model represent a random sample from a population with mean μ and variance σ_m^2 , then, even without making the assumption of homoscedasticity, we know that

$$(2.5) \qquad \hat{\sigma}_m^2 = (J - 1)\left(N - \frac{\Sigma n_j^2}{N}\right)^{-1} (MS_{between} - MS_{within})$$

is an unbiased estimator of σ_m^2 , where n_j is the number of observations in group j, N is the total number of observations in the J groups, and $MS_{between}$ and MS_{within} are the mean sums of squares in an unbalanced fixed effects analysis of variance.

In applying the theorem to the problem at hand, the estimation of σ_p^2 for use in Equation 2.4 to estimate $\underline{n}'$ and $\underline{r}'$, it helps to note that $X_{ij}^2 = X_{ij}$ for 0-1 data. The sums and sums of squares required for the

analysis of variance reduce to counts:

(2.6) $$\sum_i X_{ij}^2 = \sum_i X_{ij} = r_j \text{ , the number of successes in the } j^{th} \text{ group;}$$

$$\sum_{i,j} X_{ij}^2 = \sum_{i,j} X_{ij} = R \text{ , the total number of successes.}$$

We use Equation 2.6 to obtain a convenient computational formula for $\hat{\sigma}_p^2$.

(2.7) $$\hat{\sigma}_p^2 = \frac{\frac{N(N-1)}{N-J} \sum_j \frac{r_j^2}{n_j} - R^2 - \frac{J-1}{N-J} R}{N^2 - \sum_j n_j^2} \ .$$

In summary, we propose to estimate similar proportions in J different groups as follows. First, we estimate the mean of the distribution of the p parameters generating the proportions by $p_{\cdot} = R/N$ and the variance by Equation 2.7 . Then we find the parameters of the beta distribution having that mean and variance via Equation 2.4 . Adopting this beta distribution as our prior, we apply Equation 2.2 successively to the sample results for each group to obtain the posterior expected proportions for the respective groups.

In the case of equal sample sizes, the approach just described coincides with one proposed by Griffin and Krutckoff (1971). The proposed weighted estimator of σ_p^2 for the extension of the method to the case of unequal sample sizes is potentially a weak link in the procedure. One problem is that it can yield negative estimates of variance. When this occurs, we assume σ_p^2 is zero and regress the estimates of the proportions for all groups to the overall proportion of success. It will be argued below that in the context of estimating the variance of proportions between groups the proposed estimator is

less subject to this problem than unweighted estimators that have been suggested and is satisfactory for the use intended here. This is not to say that it is the best possible estimator. The problem of estimating the between group variance component is very complicated in the unbalanced case. Tukey (1957) has studied the variance of a family of estimators based on a very general weighting scheme that includes the weighted and unweighted estimators discussed here as special cases. Which of the latter has smaller variance depends in a complicated way on several factors. Other estimators in the family can be devised which are better than either of these under some circumstances. A single "best" estimator does not exist. The situation seems to call for the utilization of prior information about the distribution which Bayesian procedures provide for. Hill (1965) has studied such procedures under normal distribution assumptions. Unfortunately, these procedures are cumbersome to apply and must invoke assumptions and approximations to simplify the mathematics which make one question how much one would gain by employing them. The legitimate consideration of ease of computation will often limit the possibilities seriously entertained to the simple weighted and unweighted estimators. As has already been indicated, reasons for preferring the former in the context of the present problem will be presented below in the discussion of application of the proposed procedure.

Before discussing applications of the approach to estimation of proportions in several groups described in this section, let us describe Jackson's (1972) classical Model II solution to the problem. This approach is similar in spirit to the one proposed here, but somewhat more complicated to carry out.

3. An Improved Classical Solution

The main way in which the classical Model II approach to the problem of estimating proportions in several groups differs from the approach just described is that it first transforms the proportions via the root arcsine transformation

$$g_j = \sin^{-1}\left(\frac{r_j + 3/8}{n_j + 3/4}\right)^{\frac{1}{2}} . \tag{3.1}$$

The steps carried out above on the proportions themselves are carried out on the transformed proportions in the classical approach. The transformed proportions have the desirable property of being approximately normally distributed with known variances. The approximate mean and variance of g_j are given by

$$E(g_j) = \sin^{-1}\sqrt{\pi_j} = \gamma_j \tag{3.2}$$

$$\mathrm{Var}(g_j) = \frac{1}{4n_j + 2} ,$$

where π_j is the true proportion in group j. Thus, the variance of a transformed sample proportion depends only on the sample size, not on the true value of the proportion. Let μ_γ and σ_γ^2 be the mean and variance of the population of transformed true proportions from which the γ_j's are considered to have been sampled. These are estimated from the data by methods to be discussed below. Then assuming a normal prior with these parameter values,

the posterior means of the transformed proportions are

$$\hat{\gamma}_j = \frac{g_j\ \hat{\sigma}_\gamma^2 + \hat{\mu}_\gamma\ (4n_j + 2)^{-1}}{\hat{\sigma}_\gamma^2 + (4n_j + 2)^{-1}} \ . \tag{3.3}$$

When the $\hat{\gamma}_j$'s have been obtained, the estimated proportions themselves are given by the inverse transformation

$$\hat{\pi}_j = \sin^2 \hat{\gamma}_j \ . \tag{3.4}$$

Jackson (1972) estimates the mean and variance of the transformed parameters by

$$\tilde{\mu}_\gamma = \sum_j g_j / J$$

and

$$\tilde{\sigma}_\gamma^2 = (J - 1)^{-1} \sum_j (g_j - \hat{\mu}_\gamma)^2 - J^{-1} \sum_j (4n_j + 2)^{-1} \ . \tag{3.5}$$

That is, he uses the <u>unweighted</u> mean of the transformed sample proportions as the estimator of the mean, and the difference between the sample variance of the observed g_j's and the mean of their theoretical variances as the estimator of the variance of the parameter γ . We will use the following <u>weighted</u> estimators instead:

$$\hat{\mu}_\gamma = (4N + 2J)^{-1} \sum_j (4n_j + 2) g_j = g. \ , \tag{3.6}$$

$$\hat{\sigma}_\gamma^2 = \left(4N + 2J - \frac{\sum_j (4n_j + 2)^2}{4N + 2J}\right)^{-1} \left(\sum_j (4n_j + 2)(g_j - g.)^2 - (J - 1)\right) \ .$$

The estimator $\hat{\mu}_\gamma$ weights each g_j inversely to its sampling variance. For each j, the standardized variable

$$Z_g = (4n_j + 2)^{½} (g_j - \gamma_j)$$

is approximately normal in distribution, with mean 0 and variance 1. It follows that

$$Z_g^2 = (4n_j + 2) (g_j - \gamma_j)^2$$

is approximately distributed as a chi-squared variate with one degree of freedom.

If there is no variation in the γ_j's, then the statistic

$$H = \sum_j (4n_j + 2) (g_j - g.)^2$$

is approximately distributed as chi-squared with J-1 degrees of freedom. In general, its expected value is given by

$$E(H) = J - 1 + \left(4N + 2J - \frac{\sum_j (4n_j + 2)^2}{4N + 2J}\right) \sigma_\gamma^2 .$$

The latter relation is the reason we suggest the estimator $\hat{\sigma}_\gamma^2$ defined in Equation 3.5 . The weighted estimators in Equation 3.5 reduce to the estimators proposed by Jackson (1972) when the sample sizes are equal. They have much less sampling variability than the unweighted estimators when the sample sizes are grossly unequal and σ_γ^2 is relatively small, because deviations from the overall proportion based on small samples, which tend to be large, are given less weight than the more stable proportions based on larger samples. It will be seen below that the weighted estimators can yield quite different results than the unweighted estimators.

4. Reevaluation of Some Applications

Novick, Lewis, and Jackson (1973) present detailed analyses of four applications drawn from educational settings. The last example in their discussion is critical because it is the one in which the classical empirical Bayes approach and their purely Bayesian approach appear to be most clearly contrasted. The data concern the performance of students at an Eastern University who were admitted despite the fact that a regression analysis based on Scholastic Aptitude Test scores indicated that they would be unlikely to maintain a grade point average sufficient to complete their undergraduate program. The students were carefully selected on the basis of other variables such as demonstrated motivation and willingness to work hard over an extended period of time. The question addressed by the data is whether there are meaningful differences in success rate as a function of major field within the humanities and social sciences. The data are given in Table 1.

When one applies the variance estimator given in Eq. 3.5 to the data in Table 1, the resulting estimate of the variance of transformed proportions is -.004. Given this result, the only sensible course seems to be to assume that there is no real variation in the parameter. A purely Bayesian analysis, on the other hand, does suggest a small amount of real variation in the parameter, which Novick et al. argue is a more plausible interpretation of the data.

One might agree that the latter interpretation is the more plausible, but on the grounds that the negative estimate of parameter variance reflects the inadequacy of the unweighted estimator of σ_Y^2, rather than the inadequacy of the classical approach *per se*. The weighted estimator given in Eq. 3.6 yields an estimate of σ_Y^2 of +.006. To put this result in a familiar metric, the corresponding chi-squared test statistic for testing the homogeneity of the transformed proportions is 20.93, which almost reaches the .05 level critical value of 21.03. In fact, the chi-squared statistic calculated on the untransformed proportions

Table 1. Comparison of different estimates of success rate as a function of major field.*

Major Field	n_j	n_{j+}	p_j	B	$\hat{p}_j$	C′	C
1	10	5	.50	.68	.66	.66	.70
2	8	6	.75	.69	.71	.71	.70
3	72	39	.54	.64	.59	.60	.70
4	5	4	.80	.70	.71	.71	.70
5	25	13	.52	.71	.62	.63	.70
6	13	11	.85	.71	.74	.73	.70
7	81	63	.77	.72	.76	.75	.70
8	97	72	.74	.71	.73	.73	.70
9	21	14	.67	.69	.69	.69	.70
10	80	62	.78	.72	.75	.75	.70
11	11	7	.64	.69	.69	.69	.70
12	6	5	.83	.70	.72	.72	.70
13	8	5	.63	.69	.69	.69	.70

*Data adapted from Novick, Lewis, and Jackson (1973), Table VII.
Column B gives Bayesian Model II estimates for the prior specified by Novick et al. (1973).
Column $\hat{p}_j$ gives the beta binomial estimates described in Section 2.
Column C′ gives the estimates using the revised version of Jackson's (1972) classical method, with weighted estimates of μ_γ and σ_γ^2 .
Column C gives the classical estimates using unweighted estimators of μ_γ and σ_γ^2 .

$\bar{p} = .70$, $\hat{\sigma}_p^2 = .006$ (weighted estimates).

$\bar{\gamma} = .99$, $\hat{\sigma}_\gamma^2 = .006$ (weighted estimates).

is 21.36, which would lead one to reject homogeneity at the .05 level. Estimates of parameter variability close to 0 should indicate that the observed variability between samples is close to the expected value, given homogeneous parameters, which is patently not the case with these data. The sample sizes are extremely variable in this example, with the four largest samples having 72 or more cases each and the four smallest having 8 cases or less. Any adequate estimator of the variation between groups must take this variation of sample size into account.

The weighted estimator of the variance of σ_{γ}^2 will yield negative estimates only when the variation between sample proportions is such that the chi-squared statistic for testing equality of the proportions is less than its expected value under the null hypothesis. This is analogous to a F-statistic less than 1 in testing equality of means in an analysis of variance. Under these circumstances, the sensible thing to do is usually to use the overall proportion as the common estimator for all of them. To do so is <u>not</u> inconsistent with the belief that there must be <u>some</u> real differences between the parameters. It merely indicates that either the samples are so small or the sample proportions so close to one another, or both, that distinct estimates for each proportion would be misleading regarding the differences.

Both the weighted and unweighted estimators of σ_{γ}^2 can yield negative results. In order to make sure that it is not just the luck of the draw that makes the weighted estimator come out better in this case, one hundred replications mimicking the situation in this data set were simulated on a computer. In each replication, γ's having mean 1.00 and variance .006 were generated for thirteen groups using a random normal number generator. The γ for each group was then modified by a random normal sampling error with mean 0 and variance $(4n_j + 2)^{-1}$. Then weighted and unweighted estimates of σ_{γ}^2 were calculated. The distribution of the results is summarized in Table 2.

Table 2. Distribution of weighted and unweighted estimates of σ_Y^2 in 100 simulated replications under conditions represented in Table 1.

Estimated σ_Y^2	Weighted Estimates	Unweighted Estimates
est. $\sigma_Y^2 \leq .000$	10	35
$.000 <$ " $\leq .003$	24	11
$.003 <$ " $\leq .006$	20	8
$.006 <$ " $\leq .009$	20	11
$.009 <$ " $\leq .012$	17	10
$.012 <$ "	9	25
	100	100
mean	.00575	.00567
standard deviation	.00527	.01110

The unweighted estimates are negative 35 times out of 100, whereas the weighted estimates are negative only 10 times in 100. The unweighted estimates are more than double the actual value of σ_γ^2 25 times compared to only 9 times for the weighted estimates. Clearly, under the conditions present in this data set, namely large variation in sample size and small variation in γ, the weighted estimator is much better behaved than the unweighted estimator. These remarks apply with equal force to the comparison of the weighted and unweighted estimators of σ_p^2 in the direct beta-binomial method described in Section 2. The γ's in the simulations just described were transformed to p's via the relation $p=\sin^2\gamma$, and the resulting p's used to generate binary data for each group. The unweighted estimator of σ_p^2 yielded negative estimates 35 times, whereas the weighted estimator in Eq. 2.11 did so only 12 times. The standard deviation of the weighted estimator was less than half that of the unweighted estimator.

The estimates of success rate by major field produced by the different estimators are compared in Table 1. All the estimates are sharply regressed toward the overall success rate of .70, with the estimates of Jackson's (1972) method entirely regressed to .70. The contrast between the different estimators is clearest for Majors 3 and 5, whose observed success rates of .54 and .52 based on moderately large samples represent noteworthy deviations from the overall success rate. The critical ratios for these deviations from a hypothetical rate of .70 are -2.93 and -1.96 respectively. For Major 3 the empirical Bayes estimates using weighted estimates of parameter variation are .59 for the direct beta-binomial method and .60 for Jackson's method; for Major 5 these methods yield .62 and .63, respectively. There is little to choose between the direct beta-binomial method and the revised version of Jackson's method in this set of data, since the two approaches never differ by more than one point in the second decimal place. The estimates produced by either method seem more plausible for Majors 3 and 5 than .70.

The data set just discussed is well suited for showing the weighted estimators at their best because the sample sizes are so variable and the apparent variation in the underlying proportions so small. Let us turn now to an example more likely to show the unweighted estimators at their best. The first data set discussed by Novick, Lewis, and Jackson (1973) involved the analysis of the proportions of students attaining the 70th percentile on the Iowa Test of Educational Development composite score in eleven Midwestern high schools. Novick et al. consider several possible specifications of prior for σ_γ^2 , the one with largest variance having mean equal to .035 . They argue that this is about as large as it can be and still be consistent with the constraints imposed by the desire that the prior for σ_γ^2 have an inverse chi-square form with mode falling within the interquartile range, and the fact that the γ's are in the bounded interval $(0, \pi/2)$. The sample sizes in this example range from 14 to 30, but the second largest sample has only 21 subjects, so the variability in sample size is much less extreme than it was in the previous example. Moderate variation in sample size and large variation in the underlying parameter should make this example very favorable to unweighted estimators of σ_γ^2 and σ_p^2 .

A simulation study similar to the one done in the previous example was conducted using the sample sizes of the present example, a mean γ of .80 , and σ_γ^2 equal to .035 . The unweighted and the weighted estimates of σ_γ^2 were negative in 3 of 100 replications (the same replications, as it happened). The standard deviation and mean squared error were slightly smaller for the unweighted estimator, .018 vs .020 . The correlation between the two estimators was .974 . The results were even closer for the two estimators of σ_p^2 . The weighted estimate was negative once whereas the unweighted estimates never were. On the other hand the weighted estimator was (very) slightly less variable than the unweighted estimator.

The results of the simulation study suggest that it will make little difference whether weighted or unweighted estimators are used when there is not too much variability in sample size and the variation in the underlying proportions is substantial. The analyses of data for the example confirm this expectation. The data are given in Table 3. Estimators C, C′, and $\hat{p}$ are defined as in Table 1. When Novick et al. applied their Bayesian analysis using a diffuse prior with $\sigma_{\gamma}^2 = .035$, the posterior estimate of σ_{γ}^2 went down to .015 . The marked discrepancy between prior and posterior estimates of σ_{γ}^2 led them to try another analysis using the posterior specification of σ_{γ}^2 as if it were their prior. The results of this analysis, which is similar in spirit to the empirical Bayes approach, are given in the column headed B(t=16). The various analyses yield very similar results. The largest discrepancy between the weighted and unweighted versions of Jackson's method is .010 . Between the direct beta-binomial approach and the weighted version of Jackson's method the largest discrepancy is .005 . The largest difference between _any two_ of the methods is .014 .

In summary, the differences in performance between weighted and unweighted estimators of σ_{γ}^2 and σ_p^2 are not very significant when σ_{γ}^2 or σ_p^2 are large and the variability of sample sizes is moderate; that is, under conditions which would in general favor unweighted estimators. On the other hand, the differences in favor of weighted estimators are substantial when the sample sizes are extremely variable and σ_{γ}^2 is small. It is therefore recommended that weighted estimators be used as a general practice when applying these procedures.

The advantages of the empirical Bayes approaches over a strictly Bayesian Model II approach to the estimation of proportions in several groups are twofold: they are much easier to compute and they avoid the problem of the specification

Table 3. Comparison of analyses using different estimators of proportions of students attaining the 70th percentile on the Iowa Test of Educational Development composite score in eleven Midwestern high schools.*

School Number	n_j	n_{j+}	p_j	C	C'	$\hat{p}_j$	B(t=16)
1	15	10	.667	.555	.557	.561	.562
2	21	13	.619	.551	.552	.555	.556
3	16	5	.313	.481	.473	.468	.468
4	17	10	.588	.541	.540	.543	.543
5	15	6	.400	.502	.496	.493	.494
6	17	11	.647	.554	.556	.559	.560
7	18	9	.500	.521	.518	.518	.518
8	20	15	.750	.585	.591	.595	.599
9	30	12	.400	.488	.478	.474	.474
10	19	8	.421	.502	.496	.493	.494
11	14	7	.500	.522	.519	.519	.520

*Data adapted from Novick, Lewis, and Jackson (1973), Table IV.

of a prior distribution. The discussion of the latter problem by Novick, Lewis, and Jackson (1973) shows that it can be a rather delicate matter. An objection to the empirical Bayes methods is that the estimators of the variance of the underlying parameter are so poor that they cannot be trusted. The examples presented here suggest that the use of weighted estimators reduces the force of this objection considerably.

The weighted estimators are by no means perfect and in some other contexts an investigator might be well advised to use a Bayesian estimator enabling him to incorporate his prior beliefs about the extent of variation into his estimate. This might be the case, for example, if the data represent the results of a pilot study to determine the size of sample required to estimate the between group variance component accurately. The practice of setting a negative estimate of variance to zero could lead to a rather misleading conclusion in such a situation. However, in the present context, setting a negative estimate of variance to zero leads to the sensible decision to regress of estimates of proportions to the overall proportion. For present purposes, the weighted estimators proposed in this paper seem to be adequate.

The beta-binomial method of Section 2 and Jackson's classical method yield practically identical results when weighted estimators of σ_p^2 and σ_γ^2 are used, at least for the examples presented here. This is not too surprising since the root arcsine transformation is a very weak transformation of proportions falling in the broad middle range of the unit interval, as is the case with the examples discussed here. In this range it in effect just adds the constant $\pi/4 - \frac{1}{2} = .29$ to each proportion, where the constant is the difference between the midrange of transformed unit interval and the midrange of unit interval itself. For example, the interval (.20, .80) is transformed to the interval (.46, 1.11) by the root arcsine transformation and to (.49, 1.09) by adding the constant .29 . Even for the interval (.10, .90), the transformed

interval (.32, 1.25) is not very different from the shifted interval (.39, 1.19). The root arcsine transformation has a substantial effect on the spacing of proportions only when they are concentrated near zero or one. In such cases Jackson's method, using the Freeman and Tukey (1950) modification of the arcsine transformation

$$g = 1/2 \left(\sin^{-1}\sqrt{\frac{r}{n+1}} + \sin^{-1}\sqrt{\frac{r+1}{n+1}} \right) ,$$

may yield better results than the beta-binomial method. However, for problems where most of the proportions are between .10 and .90 the direct beta-binomial approach would be preferred because of its computational and conceptual simplicity.

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